CALCULUS AB

A CALCULATOR **CANNOT BE USED ON PART A OF SECTION I**. A GRAPHING CALCULATOR FROM THE APPROVED LIST **IS REQUIRED ON PART B OF SECTION I AND FOR SECTION II** OF THE EXAMINATION. CALCULATOR MEMORIES NEED NOT BE CLEARED. COMPUTERS, NONGRAPHING SCIENTIFIC CALCULATORS, CALCULATORS WITH QWERTY KEYBOARDS, AND ELECTRONIC WRITING PADS ARE NOT ALLOWED. CALCULATORS MAY NOT BE SHARED AND COMMUNICATION BETWEEN CALCULATORS IS PROHIBITED DURING THE EXAMINATION. ATTEMPTS TO REMOVE TEST MATERIALS FROM THE ROOM BY ANY METHOD WILL RESULT IN THE INVALIDATION OF TEST SCORES.

SECTION I

Time — 1 hour and 45 minutes

All questions are given equal weight.

Percent of total grade — 50

Part A: 55 minutes, 28 multiple-choice questions
A calculator is NOT allowed.

Part B: 50 minutes, 17 multiple-choice questions
A graphing calculator is required.

Parts A and B of Section I are printed in this examination booklet; Section II, which consists of longer problems, is printed in a separate booklet.

General Instructions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE INSTRUCTED TO DO SO.

INDICATE YOUR ANSWERS TO QUESTIONS IN PART A ON PAGE 2 OF THE SEPARATE ANSWER SHEET. THE ANSWERS TO QUESTIONS IN PART B SHOULD BE INDICATED ON PAGE 3 OF THE ANSWER SHEET. No credit will be given for anything written in this examination booklet, but you may use the booklet for notes or scratchwork. After you have decided which of the suggested answers is best, COMPLETELY fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

Example: Sample Answer

What is the arithmetic mean of the numbers 1, 3, and 6 ? Ⓐ Ⓑ Ⓒ ● Ⓔ

(A) 1

(B) $\frac{7}{3}$

(C) 3

(D) $\frac{10}{3}$

(E) $\frac{7}{2}$

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. In this section of the examination, as a correction for haphazard guessing, one-fourth of the number of questions you answer incorrectly will be subtracted from the number of questions you answer correctly. It is improbable, therefore, that mere guessing will improve your score significantly; it may even lower your score, and it does take time. If, however, you are not sure of the best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices as wrong, your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

Use your time effectively, working as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult. Go on to other questions and come back to the difficult ones later if you have time. It is not expected that everyone will be able to answer all the multiple-choice questions.

CALCULUS AB

SECTION I, Part A

Time — 55 minutes

Number of questions — 28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

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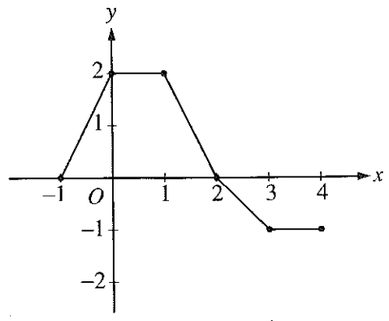

2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

3. $\int_{1}^{2} \frac{1}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) 2 ln 2

GO ON TO THE NEXT PAGE

4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

(E) $\int_{a}^{b} f(x)\,dx$ exists.

5. $\int_{0}^{x} \sin t\,dt =$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

GO ON TO THE NEXT PAGE

6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

7. $\int_1^e \left(\frac{x^2 - 1}{x}\right) dx =$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

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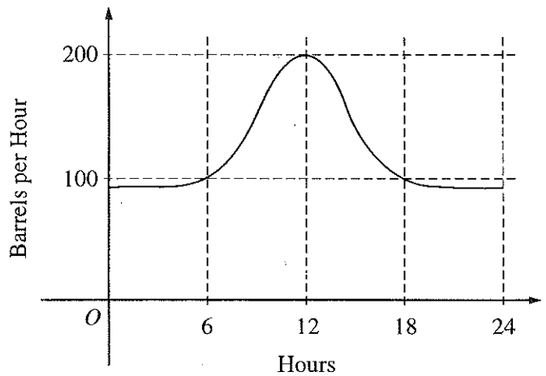
8. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

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10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

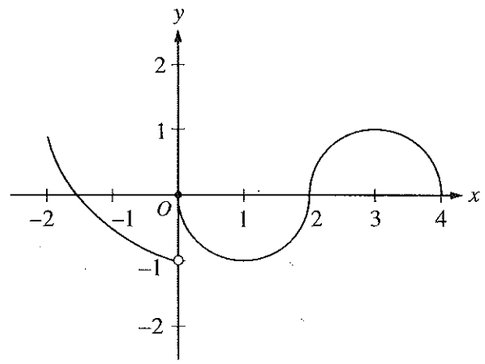
(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

11. If f is a linear function and $0 < a < b$, then $\displaystyle\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim\limits_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, –1) and (3, 1). For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

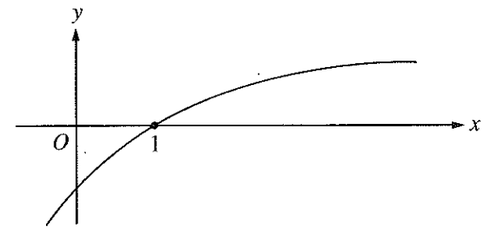
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

15. If $F(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

(A) –3 (B) –2 (C) 2 (D) 3 (E) 18

16. If $f(x) = \sin(e^{-x})$, then $f'(x) =$

(A) $-\cos(e^{-x})$
(B) $\cos(e^{-x}) + e^{-x}$
(C) $\cos(e^{-x}) - e^{-x}$
(D) $e^{-x}\cos(e^{-x})$
(E) $-e^{-x}\cos(e^{-x})$



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0, 1)$ is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

19. If $f''(x) = x(x + 1)(x - 2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

20. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$?

(A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) -3, 0, and 3

21. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

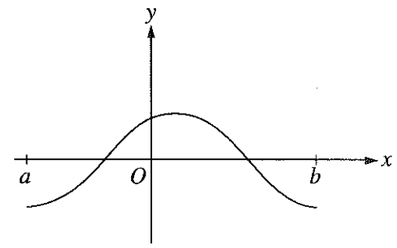
(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

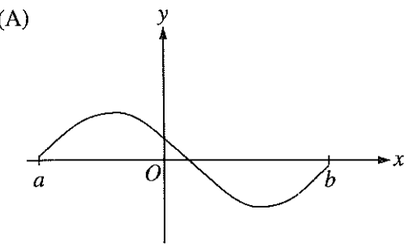
(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

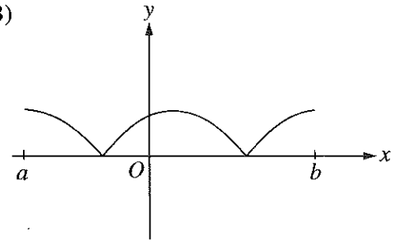


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

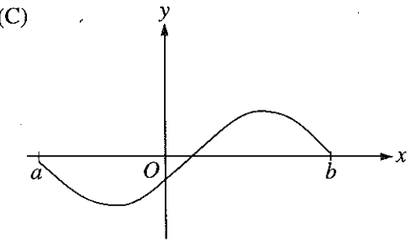
(A)



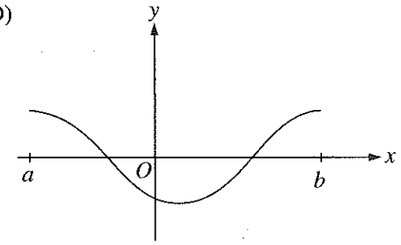
(B)



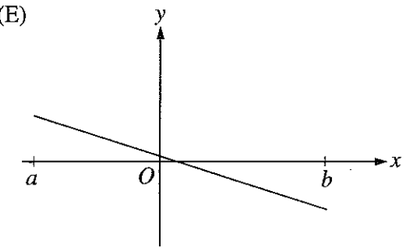
(C)



(D)



(E)



24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$

x	0	1	2
$f(x)$	1	k	2

26. The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

27. What is the average value of $y = x^2\sqrt{x^3 + 1}$ on the interval [0, 2] ?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS PART ONLY.
DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time — 50 minutes

Number of questions — 17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

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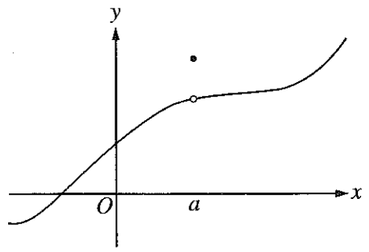


76. The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.

77. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

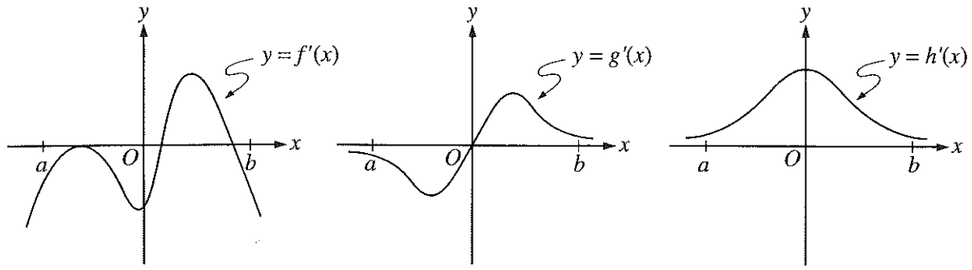
(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\dfrac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$



79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

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80. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0, 10)$?

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

81. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$.
II. f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

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82. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)dx =$

(A) $2F(3) - 2F(1)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

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83. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

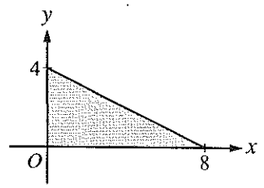
84. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210



86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

88. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

91. Let f be a function that is differentiable on the open interval $(1, 10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c) = 3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II and III

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92. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART B ONLY.
DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET AND HAVE WRITTEN AND GRIDDED YOUR AP NUMBER IN THE APPROPRIATE SECTION OF YOUR ANSWER SHEET.

AFTER TIME HAS BEEN CALLED, ANSWER QUESTIONS 93-96.

93. Which graphing calculator did you use during the examination?

(A) Casio 6300, Casio 7000, Casio 7300, Casio 7400, or Casio 7700
(B) Texas Instruments TI-80 or TI-81
(C) Casio 9700, Casio 9800, Casio 9850, Sharp 9200, Sharp 9300, Texas Instruments TI-82, Texas Instruments TI-83, Texas Instruments TI-85, or Texas Instruments TI-86
(D) Hewlett-Packard HP-48 series or HP-38G
(E) Some other calculator

94. During your Calculus AB course, which of the following best describes your calculator use?

(A) I used my own graphing calculator.
(B) I used a graphing calculator furnished by my school, both in class and at home.
(C) I used a graphing calculator furnished by my school only in class.
(D) I used a graphing calculator furnished by my school mostly in class, but occasionally at home.
(E) I did not use a graphing calculator.

95. During your Calculus AB course, which of the following describes approximately how often a graphing calculator was used by you or your teacher in classroom learning activities?

(A) Almost every class
(B) About three-quarters of the classes
(C) About one-half of the classes
(D) About one-quarter of the classes
(E) Seldom or never

96. During your Calculus AB course, which of the following describes approximately how often you were allowed to use a graphing calculator on tests?

(A) Almost all of the time
(B) About three-quarters of the time
(C) About one-half of the time
(D) About one-quarter of the time
(E) Seldom or never

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight. The problems are printed in the booklet and in the green insert; it may be easier for you to first look over all problems in the insert. When you are told to begin, open your booklet, carefully tear out the green insert, and start to work.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X², X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

General instructions for this section are printed on the back cover of this booklet.

1. Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

   (a) Find the area of the region R.

   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

GO ON TO THE NEXT PAGE

Advanced Placement Program®
THE COLLEGE BOARD

Q. THIS SECTION IS FOR THE SURVEY QUESTIONS IN THE CANDIDATE PACK. (DO NOT PUT RESPONSES TO EXAM QUESTIONS IN THIS SECTION.) BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE OVAL.

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2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

DO NOT COMPLETE THIS SECTION UNLESS INSTRUCTED TO DO SO.

R. If this answer sheet is for the French Language, French Literature, German Language, Spanish Language, or Spanish Literature Examination, please answer the following questions. (Your responses will not affect your grade.)

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken? ○ Yes ○ No
2. Do you regularly speak or hear the language at home? ○ Yes ○ No

INDICATE YOUR ANSWERS TO THE EXAM QUESTIONS IN THIS SECTION. IF A QUESTION HAS ONLY FOUR ANSWER OPTIONS, DO NOT MARK OPTION (E). YOUR ANSWER SHEET WILL BE SCORED BY MACHINE. USE ONLY NO. 2 PENCILS TO MARK YOUR ANSWERS ON PAGES 2 AND 3 (ONE RESPONSE PER QUESTION). AFTER YOU HAVE DETERMINED YOUR RESPONSE, BE SURE TO COMPLETELY FILL IN THE OVAL CORRESPONDING TO THE NUMBER OF THE QUESTION YOU ARE ANSWERING. STRAY MARKS AND SMUDGES COULD BE READ AS ANSWERS, SO ERASE CAREFULLY AND COMPLETELY. ANY IMPROPER GRIDDING MAY AFFECT YOUR GRADE.

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75 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

FOR QUESTIONS 76-151, SEE PAGE 3.

DO NOT WRITE IN THIS AREA.

G. What language do you know best?

- ○ English
- ○ English and another language about the same
- ○ Another language

H. DO NOT COMPLETE THIS SECTION UNLESS INSTRUCTED TO DO SO.

Essay Choices
Fill in the ovals under the numbers of the essay questions you answered in this examination.

1 2 3 4 5 6 7 8 9 10 11 12
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

SCHOOL USE ONLY

Fee Reduction Granted	Report to Teachers Section Designation
○	⓪ ① ② ③ ④ ⑤ ⑥ ⑦

F. AP EXAMINATION TO BE TAKEN USING THIS ANSWER SHEET

Print examination name: _____

Fill in the appropriate oval below for examination name and number.

07 ○	U.S. History	55 ○	German Language
13 ○	Art: History of	57 ○	Gov. & Pol.: **U.S.**
14 ○	Art: Studio Drawing	58 ○	Gov. & Pol.: **Comp.**
15 ○	Art: Studio General	59 ○	International English Language
20 ○	Biology	60 ○	Latin: Vergil
25 ○	Chemistry	60 ○	Latin: Literature
31 ○	Computer Science A	66 ○	Calculus AB
33 ○	Computer Science AB	68 ○	Calculus BC
34 ○	Economics: **Micro**	75 ○	Music Theory
35 ○	Economics: **Macro**	78 ○	Physics B
36 ○	Eng. Language & Comp.	80 ○	Physics C: **Mech.**
37 ○	Eng. Literature & Comp.	80 ○	Physics C: **E & M**
40 ○	Environmental Science	85 ○	Psychology
43 ○	European History	87 ○	Spanish Language
48 ○	French Language	89 ○	Spanish Literature
51 ○	French Literature	90 ○	Statistics

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P. STUDENT SEARCH SERVICE OF THE COLLEGE BOARD
(Complete ONLY if you are a SOPHOMORE or a JUNIOR.)

- ○ Yes, I want the College Board to send information about me to colleges, universities, and governmental scholarship programs interested in students like me.
- ○ No, I do not want the College Board to send information about me to colleges, universities, and governmental scholarship programs through the Student Search Service.

Q. Will you be applying for Sophomore Standing at college? ○ Yes ○ No

ETS USE ONLY

Exam	Exam
① ②	① ②
—	—
⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

Year of Birth
○ 1984 ○ 1983 ○ 1982 ○ 1981 ○ Before 1981 ○ Unstated

Ethnic Group (Race)
- ① American Indian or Alaska Native
- ② Black or African American
- ③ Mexican American or Chicano
- ④ Asian, Asian American, or Pacific Islander
- ⑤ Puerto Rican
- ⑥ South American, Latin American, Central American, or other Hispanic
- ⑦ White
- ⑧ Other

Sex ○ Female ○ Male

Test Time
○ 8 AM ○ 9 AM ○ 10 AM ○ 11 AM ○ 12 PM ○ 1 PM ○ 2 PM ○ 3 PM ○ 4 PM ○ 5 PM

Chapter III Answers to the 1998 AP Calculus AB and Calculus BC Examinations

- Section I: Multiple Choice
  - Blank Answer Sheet
- Section II: Free Response
  - Student Preparation for the Exams
  - Free-Response Questions, Scoring Guidelines, and Sample Student Responses with Commentary
  - Section II, Calculus AB
  - Section II, Calculus BC

Section I: Multiple Choice

Listed below are the correct answers to the multiple-choice questions and the percentage of AP candidates who answered each question correctly. A copy of the blank answer sheet appears on the following pages for reference.

Section 1 Answer Key and Percent Answering Correctly
Calculus AB

Item No.	Correct Answer	Percent Correct by Grade 5	4	3	2	1	Total Percent Correct
1	D	96%	92%	83%	70%	41%	78%
2	B	75%	55%	37%	24%	19%	43%
3	C	94%	85%	75%	60%	38%	71%
4	B	82%	62%	47%	36%	31%	52%
5	E	78%	64%	50%	39%	26%	52%
6	A	94%	86%	75%	53%	21%	68%
7	E	85%	59%	38%	22%	13%	43%
8	E	77%	53%	43%	37%	31%	48%
9	D	90%	77%	65%	55%	45%	67%
10	D	91%	83%	74%	62%	62%	74%
11	A	76%	50%	36%	27%	17%	42%
12	E	91%	71%	48%	29%	20%	52%
13	B	72%	55%	41%	31%	20%	46%
14	C	100%	99%	98%	94%	74%	94%
15	D	89%	83%	76%	59%	39%	71%
16	E	94%	87%	74%	54%	29%	69%
17	D	74%	44%	24%	12%	7%	33%
18	B	96%	86%	72%	54%	35%	71%
19	C	55%	36%	23%	17%	10%	28%
20	A	88%	83%	71%	62%	39%	69%
21	B	59%	34%	25%	20%	23%	31%
22	C	93%	84%	69%	57%	39%	69%
23	A	94%	85%	70%	51%	30%	68%
24	D	65%	54%	51%	51%	39%	52%
25	D	94%	83%	75%	62%	36%	71%
26	A	80%	51%	31%	21%	16%	41%
27	A	72%	50%	33%	21%	13%	40%
28	E	86%	71%	51%	32%	9%	56%
76	A	79%	65%	59%	56%	49%	61%
77	C	96%	82%	63%	41%	25%	63%
78	B	87%	66%	42%	22%	9%	46%
79	A	84%	66%	46%	27%	11%	47%
80	B	72%	57%	46%	34%	26%	48%
81	D	93%	82%	73%	66%	51%	74%
82	E	47%	28%	19%	15%	14%	24%
83	B	81%	60%	40%	28%	19%	45%
84	A	82%	57%	38%	21%	11%	42%
85	C	80%	56%	42%	29%	21%	46%
86	C	33%	15%	12%	17%	23%	19%
87	D	80%	50%	32%	20%	17%	40%
88	C	89%	71%	54%	38%	24%	55%
89	B	76%	48%	31%	19%	17%	38%
90	D	70%	45%	30%	20%	15%	36%
91	E	80%	64%	49%	34%	18%	51%
92	D	97%	91%	80%	62%	30%	76%

Section 1 Answer Key and Percent Answering Correctly
Calculus BC

Item No.	Correct Answer	Percent Correct by Grade 5	4	3	2	1	Total Percent Correct
1	C	91%	81%	79%	67%	55%	80%
2	A	98%	96%	92%	85%	72%	92%
3	D	80%	63%	50%	30%	26%	58%
4	A	83%	65%	49%	32%	19%	61%
5	A	84%	64%	51%	35%	20%	60%
6	E	93%	89%	84%	69%	50%	83%
7	E	89%	74%	61%	41%	31%	68%
8	B	78%	60%	41%	28%	13%	55%
9	D	91%	83%	76%	67%	58%	80%
10	E	90%	78%	72%	64%	39%	75%
11	A	76%	57%	44%	34%	18%	55%
12	E	90%	77%	64%	49%	27%	72%
13	B	74%	66%	58%	45%	29%	62%
14	E	86%	73%	62%	43%	31%	68%
15	B	89%	80%	71%	63%	41%	76%
16	C	55%	40%	29%	25%	15%	37%
17	D	77%	59%	41%	24%	11%	54%
18	B	57%	36%	23%	21%	13%	35%
19	D	56%	34%	27%	17%	13%	37%
20	E	79%	56%	39%	32%	9%	53%
21	C	89%	78%	66%	46%	38%	73%
22	A	83%	69%	59%	56%	47%	68%
23	E	53%	28%	21%	14%	8%	31%
24	C	60%	38%	24%	17%	12%	38%
25	C	68%	44%	33%	16%	17%	45%
26	E	32%	16%	10%	12%	13%	20%
27	D	49%	29%	27%	16%	17%	35%
28	C	72%	47%	46%	33%	25%	56%
76	D	86%	64%	47%	34%	22%	60%
77	E	94%	90%	85%	83%	70%	87%
78	B	88%	76%	68%	41%	28%	69%
79	A	57%	47%	33%	33%	22%	45%
80	B	96%	92%	93%	86%	69%	91%
81	B	43%	21%	16%	10%	7%	25%
82	B	72%	44%	38%	28%	16%	48%
83	C	44%	24%	13%	12%	11%	27%
84	B	63%	38%	27%	20%	12%	40%
85	C	78%	58%	45%	38%	24%	58%
86	C	50%	26%	19%	11%	17%	32%
87	D	79%	56%	42%	35%	21%	57%
88	C	93%	83%	66%	48%	34%	75%
89	A	74%	55%	46%	37%	27%	56%
90	A	46%	31%	26%	20%	9%	32%
91	E	37%	25%	17%	17%	14%	26%
92	D	86%	66%	46%	31%	14%	65%

Commentary:

This problem asks the student to answer the straightforward questions of finding the area of a region R and the volume of the solid of revolution formed by revolving R around the x-axis (a solid with circular cross sections). The other two parts of the problem involve finding the vertical line that divides the region R into two regions of equal area and the vertical line that divides R into two regions which generate solids of equal volume when revolved about the x-axis.

This was a relatively easy question with a mean score of 6.75.

Excellent: 9 points

The student presents all required work and answers as directed. Sketches of graphs and boxed answers remove all uncertainty.

Good: 7 points

All methods appear correct but decimal answers in parts (a) and (d) are incorrectly presented, losing 2 points. There was no need to rewrite $\frac{16}{3}$, $4^{\frac{3}{2}}$, 8π, and $\sqrt{8}$ in decimal form, but if that is done, the general instructions require accuracy to the third decimal place.

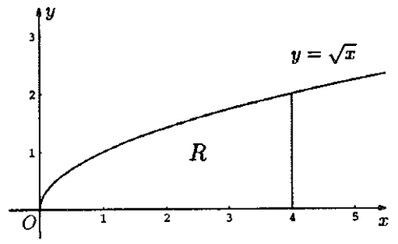
Fair: 5 points

Parts (a) and (c) are correct for the 5 out of 5 points. The student misunderstands parts (b) and (d) and gives as answers half the area and half the volume, losing 4 points.

AB–1 1998

1. Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
   (a) Find the area of the region R.
   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
   (c) Find the volume of the solid generated when R is revolved about the x–axis.
   (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

(a)



$A = \int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{16}{3}$ or 5.333

2 { 1: $A = \int_0^4 \sqrt{x}\,dx$; 1: answer

(b) $\int_0^h \sqrt{x}\,dx = \frac{8}{3}$ | –or– | $\int_0^h \sqrt{x}\,dx = \int_h^4 \sqrt{x}\,dx$

$\frac{2}{3}h^{3/2} = \frac{8}{3}$ | | $\frac{2}{3}h^{3/2} = \frac{16}{3} - \frac{2}{3}h^{3/2}$

$h = \sqrt[3]{16}$ or 2.520 or 2.519

2 { 1: equation in h ; 1: answer

(c) $V = \pi \int_0^4 (\sqrt{x})^2\,dx = \pi\frac{x^2}{2}\Big|_0^4 = 8\pi$

or 25.133 or 25.132

3 { 1: limits and constant ; 1: integrand ; 1: answer

(d) $\pi \int_0^k (\sqrt{x})^2\,dx = 4\pi$ | –or– | $\pi \int_0^k (\sqrt{x})^2\,dx = \pi \int_k^4 (\sqrt{x})^2\,dx$

$\pi\frac{k^2}{2} = 4\pi$ | | $\pi\frac{k^2}{2} = 8\pi - \pi\frac{k^2}{2}$

$k = \sqrt{8}$ or 2.828

2 { 1: equation in k ; 1: answer

Excellent Sample Student Response: 9 points

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

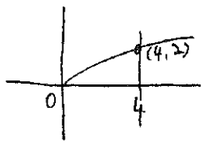
A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

General instructions for this section are printed on the back cover of this booklet.

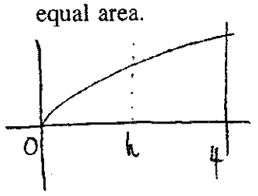
1. Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

(a) Find the area of the region R.



$$\int_0^4 \sqrt{x}\,dx = 5.33333\ldots \approx \boxed{5.333}$$

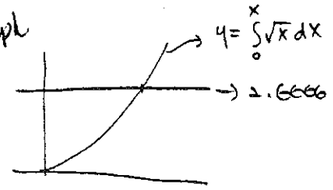
(b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.



$$\int_0^h \sqrt{x}\,dx = \frac{1}{2}\int_0^4 \sqrt{x}\cdot dx$$

$$\int_0^h \sqrt{x}\,dx = 2.666666$$

use graph



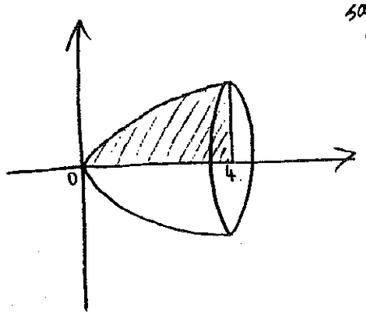
intersection : (2.5198415, 2.666)

@ X=2.5198

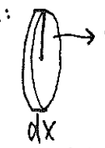
So,

$h \approx 2.520$

(c) Find the volume of the solid generated when R is revolved about the x-axis.



sample disc:



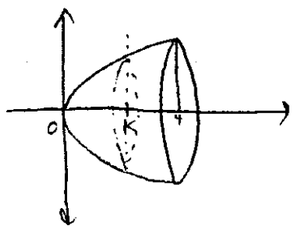
$V = \pi r^2 h$

$= \pi(\sqrt{x})^2 \cdot dx = \pi \cdot x\,dx$

$$V = \pi \int_0^4 x\,dx = 8\pi$$

Volume = $\boxed{8\pi}$

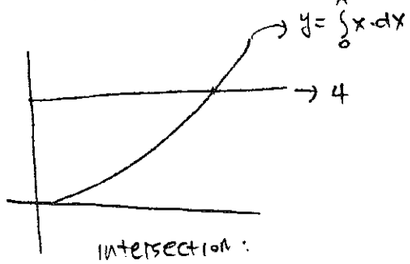
(d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.



$$\pi \int_0^k x \cdot dx = \frac{1}{2} \cdot \pi \int_0^4 x\,dx$$

$$\pi \int_0^k x\,dx = \frac{1}{2} \cdot 8\pi = 4\pi$$

use graph



intersection : (2.82842, 4)

@ X = 2.82842

So,

$k \approx 2.828$

Commentary:

In the first three parts of this common problem, the student is asked to analyze the behavior of a given function over its entire domain. In part (b), the student must give an explicit argument for why the critical point is an absolute minimum; relative arguments are not sufficient. The absolute minimum found in part (b), together with the limits in the first part, lead directly to finding the range in part (c). In the last part, the function is generalized to a family of functions and students are required to deal with the parameter b correctly to receive credit; studying a specific member of the family can help develop intuition about the problem but does not show the result for the entire family.

The mean scores were 3.76 for Calculus AB and 5.41 for Calculus BC. Students had difficulty with justification of the absolute minimum of $2xe^{2x}$ and handling bxe^{bx} for general nonzero b.

Excellent: 9 points

The student gives correct and sufficiently thorough answers in all parts although the notation in part (a) leading to the answer is flawed. In part (b) the justification is both local and global as indicated by either the boxed statement or the labeled real line, assuming that the arrows on the ends of the line are read as $f'(x)$ being negative (or positive) for all $x < -\frac{1}{2}$ (or all $x > -\frac{1}{2}$).

Good: 7 points

The student earns 2 out of 2 points in part (a) and is given 1 out of 1 point in part (c) in spite of some shaky language describing the range. In part (b) the student does not earn the point for justification because the argument is clearly local, thus earning 2 out of 3 points. The student is awarded 2 out of 3 points for part (d) although giving b as $-\frac{1}{x}$ is not exactly as prescribed by the standard for the second point. However, the important feature is that $bx = -1$.

Fair: 5 points

The student earns 2 out of 2 points in part (a); 1 out of 3 points for solving $f'(x) = 0$ in part (b); 0 out of 1 point for an incorrect range in part (c); and the first 2 out of 3 points in part (d).

2. Let f be the function given by $f(x) = 2xe^{2x}$.
   (a) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.
   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   (c) What is the range of f?
   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

(a) $\lim_{x\to-\infty} 2xe^{2x} = 0$

$\lim_{x\to\infty} 2xe^{2x} = \infty$ or DNE

2 { 1: 0 as $x \to -\infty$; 1: ∞ or DNE as $x \to \infty$ }

(b) $f'(x) = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} = 2e^{2x}(1 + 2x) = 0$ if $x = -1/2$

$f(-1/2) = -1/e$ or -0.368 or -0.367

$-1/e$ is an absolute minimum value because:

(i) $f'(x) < 0$ for all $x < -1/2$ and $f'(x) > 0$ for all $x > -1/2$

–or–

(ii)

x		$-1/2$	
$f'(x)$	$-$	0	$+$

and $x = -1/2$ is the only critical number

3 { 1: solves $f'(x) = 0$; 1: evaluates f at student's critical point; 0/1 if not local minimum from student's derivative ; 1: justifies absolute minimum value; 0/1 for a local argument; 0/1 without explicit symbolic derivative }

Note: 0/3 if no absolute minimum based on student's derivative

(c) Range of $f = [-1/e, \infty)$
or $[-0.367, \infty)$
or $[-0.368, \infty)$

1: answer

Note: must include the left–hand endpoint; exclude the right–hand "endpoint"

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$ if $x = -1/b$

At $x = -1/b$, $y = -1/e$

y has an absolute minimum value of $-1/e$ for all nonzero b

3 { 1: sets $y' = be^{bx}(1 + bx) = 0$; 1: solves student's $y' = 0$; 1: evaluates y at a critical number and gets a value independent of b }

Note: 0/3 if only considering specific values of b

(c) What is the range of f?

Range: $y \in \mathbb{R}$

(d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

$y = bxe^{bx}$

$y' = b(xe^{bx} \cdot b + e^{bx})$

$y' = b^2xe^{bx} + be^{bx}$

$y' = be^{bx}(bx + 1)$

$bx + 1 < 0$

$x = \frac{-1}{b}$

AB3 Scoring Guidelines

Commentary:

This problem illustrates one of the new topics in the course description, namely multiple representations of functions. Here, the velocity of a car is given both graphically and numerically, but never algebraically. Some of the parts require the graph, some require the table, and some can be successfully answered using either: the graph is needed for part (a), the table for part (b), either for (c), and the table for (d). The answer to part (b) is an exact answer, while the answers to (c) and (d) are approximations.

The mean score was 3.82 with a variety of mistakes spread over all parts of the question.

Excellent: 9 points

In part (a) two correct intervals are given with correct reason. In addition, concavity is discussed correctly. In part (b) the answer is correct. In part (c) a reasonable approximation is given with the computation as the slope of a secant line. In part (d) the correct Riemann sum is given with correct units and explanation.

Good: 7 points

Parts (a) and (c) are correct, earning 5 out of 5 points. In part (b) the student divides by 50 twice, losing the point and thus receiving 0 out of 1 point. In part (d) the meaning of the integral is not correct, thus 2 out of 3 points.

Fair: 5 points

The student earns 3 out of 3 points in part (a) with a description of the correct intervals and correct reason. Part (b)'s solution earns 1 out of 1 point. The method described in part (c) is correct, but the student loses a minus sign and a point. No points are earned in part (d). The explanation is correct, but correct units are not included.

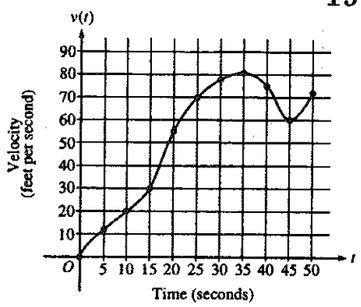
3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

   (d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

(a) Acceleration is positive on $(0, 35)$ and $(45, 50)$ because the velocity $v(t)$ is increasing on $[0, 35]$ and $[45, 50]$

3 { 1: $(0, 35)$; 1: $(45, 50)$; 1: reason

Note: ignore inclusion of endpoints

(b) Avg. Acc. $= \dfrac{v(50) - v(0)}{50 - 0} = \dfrac{72 - 0}{50} = \dfrac{72}{50}$
or 1.44 ft/sec^2

1: answer

(c) Difference quotient; e.g.

$\dfrac{v(45) - v(40)}{5} = \dfrac{60 - 75}{5} = -3$ ft/sec^2 or

$\dfrac{v(40) - v(35)}{5} = \dfrac{75 - 81}{5} = -\dfrac{6}{5}$ ft/sec^2 or

$\dfrac{v(45) - v(35)}{10} = \dfrac{60 - 81}{10} = -\dfrac{21}{10}$ ft/sec^2

–or–

Slope of tangent line, e.g.

through $(35, 90)$ and $(40, 75)$: $\dfrac{90 - 75}{35 - 40} = -3$ ft/sec^2

2 { 1: method; 1: answer

Note: 0/2 if first point not earned

(d) $\int_0^{50} v(t)\,dt$

$\approx 10[v(5) + v(15) + v(25) + v(35) + v(45)]$

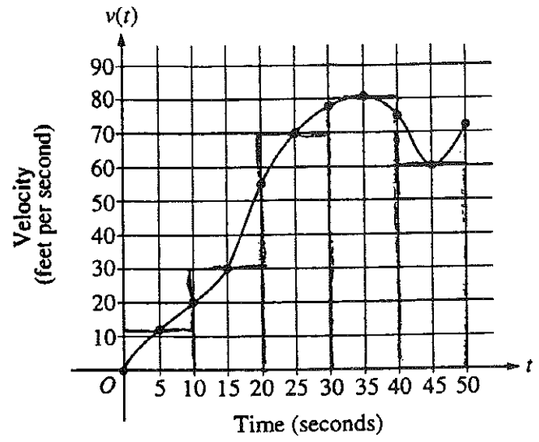
$= 10(12 + 30 + 70 + 81 + 60)$

$= 2530$ feet

This integral is the total distance traveled in feet over the time 0 to 50 seconds.

3 { 1: midpoint Riemann sum; 1: answer; 1: meaning of integral

Excellent Sample Student Response: 9 points



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

Acceleration of car is postive when the graph of position is concave up or the graph of velocity is increasing

$x''(t) = v'(t) = a(t)$ $(0, 35), (45, 50)$

(b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

$\dfrac{v(50) - v(0)}{50 - 0} = \dfrac{72 - 0}{50 - 0} = 1.44$ ft/sec^2

(c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

the average acceleration between 35 and 45 is approximately equal to that at 40s because a line going through (35, 81) and (45, 60) is close to parallel to the tangent line at t = 40.

$a_{av} = \frac{60-81}{45-35}$ $\left(a_{av} = \frac{21}{10}\right)$

(d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

this integral represents the distance carred by the car from t=0s to t = 50s.

AB4 Scoring Guidelines

Commentary:

In this problem, students are given information about the slope of the point (x, y) on a graph, which is equivalent to being given a differential equation. The exact value of $f(1.2)$ is found from the solution of the separable differential equation, which compares closely with the approximation to this value found by using the tangent line to f at $x = 1$. Unlike recent years, this differential equation is not presented in a physical context.

The mean score was 4.61. Some students were able to solve the differential equation in part (c) but did not answer the presumably easier questions in parts (a) and (b) about slope and tangent line at a particular point.

Excellent: 9 points

The student earns all points in all parts with minimal writing. In part (c) the constant of integration is introduced one line late but correctly there.

Good: 7 points

The student earns 1 out of 1 point in part (a) and 2 out of 2 points in part (b). In part (c) the student separates variables correctly and solves correctly until the final line where the branch with the point (1, 4) should be chosen. The student earns 4 out of 5 points. In part (d) the student does not find a single value for $f(1.2)$ because there is no solution in part (c); thus 0 out of 1 point.

Fair: 5 points

The student earns 1 out of 1 point in part (a). The equation of the tangent line is correct in part (b) but not used to approximate $f(1.2)$, which gives the student 1 out of 2 points. In part (c) the student separates variables and antidifferentiates both expressions for 3 out of 5 points. Part (d) is blank.

4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.

   (a) Find the slope of the graph of f at the point where $x = 1$.

   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

   (c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

   (d) Use your solution from part (c) to find $f(1.2)$.

(a) $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$

$\left.\dfrac{dy}{dx}\right|_{\substack{x=1\\y=4}} = \dfrac{3+1}{2\cdot 4} = \dfrac{4}{8} = \dfrac{1}{2}$

1: answer

(b) $y - 4 = \dfrac{1}{2}(x-1)$

$f(1.2) - 4 \approx \dfrac{1}{2}(1.2-1)$

$f(1.2) \approx 0.1 + 4 = 4.1$

2 { 1: equation of tangent line; 1: uses equation to approximate $f(1.2)$

(c) $2y\,dy = (3x^2+1)\,dx$

$\int 2y\,dy = \int (3x^2+1)\,dx$

$y^2 = x^3 + x + C$

$4^2 = 1 + 1 + C$

$14 = C$

$y^2 = x^3 + x + 14$

$y = \sqrt{x^3+x+14}$ is branch with point $(1, 4)$

$f(x) = \sqrt{x^3+x+14}$

5 { 1: separates variables; 1: antiderivative of dy term; 1: antiderivative of dx term; 1: uses $y = 4$ when $x = 1$ to pick one function out of a family of functions; 1: solves for y; 0/1 if solving a linear equation in y; 0/1 if no constant of integration

Note: max 0/5 if no separation of variables

Note: max 1/5 [1-0-0-0-0] if substitutes value(s) for x, y, or dy/dx before antidifferentiation

(d) $f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$

1: answer, from student's solution to the given differential equation in (c)

Commentary:

In the second common problem, a function representing the outside temperature for a 24-hour period is given. Students are asked to sketch the graph of the function as an aid to helping them get a feel for the question. Definite integrals are used to compute both the average temperature over an 8-hour time interval and the cost of air conditioning the house during the 24 hours. The last part of the problem is an example of using a definite integral as an accumulator. It is expected that students compute the integrals using a calculator.

The mean scores were 4.09 for Calculus AB and 5.67 for Calculus BC. Many students did not give the answers, as requested, to the nearest degree and nearest cent.

Excellent: 9 points

The student earns all points with minimal writing, evidently evaluating the two integrals and finding zeros using a calculator. Although the setups for the integrals use $F(t)$ and $g(t)$, both are defined, so the setups are correct.

Good: 7 points

The student earns 1 out of 1 point in part (a) and 3 out of 3 points in part (b). It is noted that the calculator notation (fnInt) in part (b) is not sufficient for setup, and, in the presence of the integral is superfluous. The solution is correct in part (c), but the inequality (or equality) being solved is not given, earning 1 out of 2 points. In part (d) the student loses 1 point (for constant and limits) for adding the extra factor 13.538, the length of the interval.

Fair: 5 points

The student earns 1 out of 1 point in part (a). In part (b) the student averages the two temperatures at the endpoints, earning 0 out of 3 points. In part (c) the student gives an interval, but one endpoint is not sufficiently accurate. One of the 2 points is earned, however, because the student indicates the equation being solved by a horizontal line $y = 78$ drawn on the graph in the stem of the question. In part (d) the answer and setup (using the student's endpoints from part (c)) are correct earning 3 out of 3 points.

5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right),\ 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

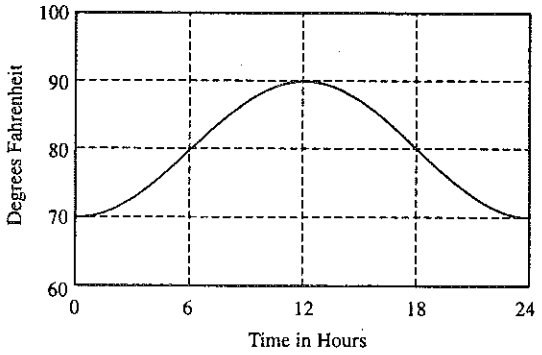
(a) Sketch the graph of F on the grid below.

(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24–hour period?

(a)



1: bell–shaped graph
minimum 70 at $t = 0$, $t = 24$ only
maximum 90 at $t = 12$ only

(b) $\text{Avg.} = \dfrac{1}{14-6}\displaystyle\int_6^{14}\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right]dt$

$= \dfrac{1}{8}(697.2957795)$

$= 87.162$ or 87.161

$\approx 87^\circ$ F

3
- 2: integral
  - 1: limits and $1/(14-6)$
  - 1: integrand
- 1: answer
  - 0/1 if integral not of the form $\dfrac{1}{b-a}\displaystyle\int_a^b F(t)\,dt$

(c) $\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right] - 78 \ge 0$

$2 - 10\cos\left(\frac{\pi t}{12}\right) \ge 0$

$\left.\begin{matrix}5.230\\ \text{or}\\ 5.231\end{matrix}\right\} \le t \le \left\{\begin{matrix}18.769\\ \text{or}\\ 18.770\end{matrix}\right.$

2
- 1: inequality or equation
- 1: solutions with interval

(d) $C = 0.05\displaystyle\int_{\substack{5.231\\ \text{or}\\ 5.230}}^{\substack{18.769\\ \text{or}\\ 18.770}}\left(\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right] - 78\right)dt$

$= 0.05(101.92741) = 5.096 \approx \$5.10$

3
- 2: integral
  - 1: limits and 0.05
  - 1: integrand
- 1: answer
  - 0/1 if integral not of the form $k\displaystyle\int_a^b (F(t) - 78)\,dt$

(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(5.3, 18.77)

(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

13.47 hours

$$\int_{5.3}^{18.77} \left(80 - 10\cos\frac{\pi t}{12}\right) - 78 \, dt$$

$$= 101.92$$

$101.92 \cdot .05 = \$5.10$

AB6 Scoring Guidelines

Commentary:

An implicitly defined function is given, and in part (a) the student is asked to verify its derivative. The derivative was given so that each student would be dealing with the correct derivative throughout the other, more complicated parts of the problem and be able to find the point P in part (c). In the second part, there is both an x-value and a y-value to be analyzed, and the y-value where the derivative is zero does not correspond to a point on the curve. In the last part, students can use either the curve or the derivative to find the point P.

The mean score on this question was 2.86 with 80 percent of the scores 4 or below and over 50 percent of the scores 2, 3, or 4. In spite of this, the scores discriminated very well at all grade levels.

Excellent: 9 points

The student earns all 9 points. In part (b) the elimination of $y = 2$ could be better presented, and the check in part (c) is prudent but not required.

Good: 7 points

The student earns 2 out of 2 points in part (a). In part (b) only the first two points are earned by solving $\frac{dy}{dx} = 0$. In part (c) the correct solution earns 3 out of 3 points.

Fair: 5 points

The student earns 2 out of 2 points in part (a). In part (b) only the first two points are earned for solving $\frac{dy}{dx} = 0$. The student earns 1 out of 3 points in part (c) for setting $\frac{dy}{dx} = -1$.

6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.
   (a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.
   (b) Write an equation of each horizontal tangent line to the curve.
   (c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x– and y–coordinates of point P.

(a) $6y^2\frac{dy}{dx} + 6x^2\frac{dy}{dx} + 12xy - 24x + 6\frac{dy}{dx} = 0$

$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$

$\frac{dy}{dx} = \frac{24x - 12xy}{6x^2 + 6y^2 + 6} = \frac{4x - 2xy}{x^2 + y^2 + 1}$

2 { 1: implicit differentiation; 1: verifies expression for $\frac{dy}{dx}$

(b) $\frac{dy}{dx} = 0$

$4x - 2xy = 2x(2 - y) = 0$

$x = 0$ or $y = 2$

When $x = 0$, $2y^3 + 6y = 1$; $y = 0.165$

There is no point on the curve with y coordinate of 2.

$y = 0.165$ is the equation of the only horizontal tangent line.

4 { 1: sets $\frac{dy}{dx} = 0$; 1: solves $\frac{dy}{dx} = 0$; 1: uses solutions for x to find equations of horizontal tangent lines; 1: verifies which solutions for y yield equations of horizontal tangent lines

Note: max 1/4 [1-0-0-0] if $dy/dx = 0$ is not of the form $g(x,y)/h(x,y) = 0$ with solutions for both x and y

(c) $y = -x$ is equation of the line.

$2(-x)^3 + 6x^2(-x) - 12x^2 + 6(-x) = 1$

$-8x^3 - 12x^2 - 6x - 1 = 0$

$x = -1/2, \quad y = 1/2$

–or–

$\frac{dy}{dx} = -1$

$4x - 2xy = -x^2 - y^2 - 1$

$4x + 2x^2 = -x^2 - x^2 - 1$

$4x^2 + 4x + 1 = 0$

$x = -1/2, \quad y = 1/2$

3 { 1: $y = -x$; 1: substitutes $y = -x$ into equation of curve; 1: solves for x and y

–or–

3 { 1: sets $\frac{dy}{dx} = -1$; 1: substitutes $y = -x$ into $\frac{dy}{dx}$; 1: solves for x and y

Note: max 2/3 [1-1-0] if importing incorrect derivative from part (a)

Table 4.3a — Grade Distributions Calculus AB

More than 66% of the candidates earned an AP grade of 3 or higher.

	Examination Grade	Total Group Number of Students	Percent at Grade
Extremely well qualified	5	18,522	15.9
Well qualified	4	27,102	23.3
Qualified	3	31,286	26.9
Possibly qualified	2	20,732	17.8
No recommendation	1	18,875	16.2

Total Number of Students	116,517
Mean Grade	3.05
Standard Deviation	1.30

Section I Scores and AP Grades

Table 4.4a — Calculus AB

This table gives the probabilities that a student would receive a particular grade on the 1998 AP Calculus AB Examination given the student's score on the multiple-choice section.

Multiple-Choice	AP Grade					Total
	1	2	3	4	5	
32 to 45	0.0%	0.0%	0.2%	11.5%	88.3%	14.1%
24 to 31	0.0%	0.2%	12.9%	71.9%	15.0%	22.7%
16 to 23	0.2%	11.9%	68.7%	19.2%	0.0%	27.8%
10 to 15	14.7%	60.4%	24.8%	0.1%	0.0%	18.9%
0 to 9	81.0%	18.4%	0.5%	0.0%	0.0%	16.5%
Total	16.2%	17.8%	26.9%	23.3%	15.9%	100.0%

Table 4.1b — Section II Scores Calculus BC

These are the score distributions for the total group of candidates on each free-response question from the 1998 exam.

	Question 1		Question 2		Question 3	
Score	Number of Candidates	% At Score	Number of Candidates	% At Score	Number of Candidates	% At Score
9	7,096	26.5	1,884	7.0	319	1.2
8	9,569	35.7	4,519	16.9	2,005	7.5
7	4,192	15.7	4,057	15.1	3,072	11.5
6	2,266	8.5	3,749	14.0	3,269	12.2
5	1,564	5.8	3,665	13.7	3,068	11.5
4	604	2.3	2,637	9.8	2,868	10.7
3	800	3.0	2,197	8.2	2,801	10.5
2	233	0.9	2,276	8.5	2,410	9.0
1	170	0.6	951	3.6	1,874	7.0
0	212	0.8	570	2.1	2,916	10.9
*NR	78	0.3	279	1.0	2,182	8.1

	Question 4		Question 5		Question 6	
Score	Number of Candidates	% At Score	Number of Candidates	% At Score	Number of Candidates	% At Score
9	1,240	4.6	4,031	15.1	1,695	6.3
8	2,681	10.0	2,257	8.4	1,719	6.4
7	2,614	9.8	3,789	14.1	2,220	8.3
6	4,217	15.7	5,444	20.3	3,465	12.9
5	3,640	13.6	3,740	14.0	3,856	14.4
4	2,103	7.9	2,274	8.5	3,680	13.7
3	1,314	4.9	2,037	7.6	3,550	13.3
2	596	2.2	1,630	6.1	2,565	9.6
1	2,939	11.0	1,108	4.1	2,229	8.3
0	3,609	13.5	222	0.8	1,010	3.8
*NR	1,831	6.8	252	0.9	795	3.0

*NR — No response. Student gave either no response or a response not on the topic.

	Question 1	Question 2	Question 3	Question 4	Question 5	Question 6
Total Candidates	26,784	26,784	26,784	26,784	26,784	26,784
Mean	7.34	5.41	3.81	4.14	5.67	4.38
Standard Deviation	1.81	2.40	2.69	2.96	2.35	2.49
Mean as % of Maximum Score	82	60	42	46	63	49

Table 4.2c — Scoring Worksheet — AP Calculus BC — AB Subscore Grade

Section I: Multiple Choice

Questions (1, 3, 5-9, 11-13, 16, 17, 20, 23, 78-82, 85-88, 90-92)

[_____ (Number correct (out of 26)) − (1/4 × _____ (Number wrong))] × 1.2692 = _____ (Multiple-Choice Score) = _____ (Weighted Section I Score)

Section II: Free Response

Question 1 _____ (out of 9) × 1.000 = _____

Question 2 _____ (out of 9) × 1.000 = _____

Question 4 (Part c) _____ (out of 6) × 1.000 = _____

Question 5 _____ (out of 9) × 1.000 = _____

Sum = _____ (Weighted Section II Score)

Composite Score

_____ (Weighted Section I Score) + _____ (Weighted Section II Score) = _____ (Composite Score (Round to the nearest whole number.))

AP Grade Conversion Chart
AB Subscore Grade

Composite Score Range*	AP Grade
42-66	5
30-41	4
19-29	3
11-18	2
0-10	1

*The candidates' scores are weighted according to formulas determined by the Development Committee to yield raw composite scores; the Chief Faculty Consultant is responsible for converting composite scores to the 5-point AP scale.

Table 4.3c — Grade Distributions Calculus BC: AB Subscore Grade

More than 90% of the candidates earned an AB subscore grade of 3 or higher.

	Examination Grade	Total Group Number of Students	Total Group Percent at Grade
Extremely well qualified	5	10,839	40.5
Well qualified	4	8,405	31.4
Qualified	3	4,960	18.5
Possibly qualified	2	1,723	6.4
No recommendation	1	857	3.2
Total Number of Students		26,784	
Mean Grade		3.99	
Standard Deviation		1.07	